

# Chapter 7

## Sediment transport model

### 7.1 Introduction

This chapter describes the sediment transport model implemented in COHERENS. There are six sections. In the first one, a discussion is given of physical parameters and processes (bed shear stress, molecular viscosity, waves) which are of importance for the sediment and the influence of sediments on the physics through density gradients. In the next section a description is given of the basic sediment parameters (Shield parameter, fall velocity, critical shear stress), related to sediment transport. This is followed by an overview of the bed and total load equations available in COHERENS. Then, the suspended sediment transport module is presented. Finally, numerical techniques, specific for the sediment module, are described.

### 7.2 Physical aspects

#### 7.2.1 Bed shear stresses

The bed shear stress is considered as the physical parameter which has the largest impact on sediment transport, since it controls how much of the sediment in the bed layer becomes suspended in the water column. In the physical part of COHERENS, bottom stresses are calculated using either a linear or a quadratic friction law (see Section 4.9). In the sediment transport module, however, a quadratic friction law is always taken, as given by equation (4.340) in 3-D or (4.341) in 2-D mode. The magnitude of the bottom stress can then be written as

$$\tau_b = \rho u_{*b}^2 = \rho C_{db} u_b^2 \quad \text{or} \quad \tau_b = \rho u_{*b}^2 = \rho C_{db} |\bar{\mathbf{u}}|^2 \quad (7.1)$$

for the 3-D, respectively 2-D case. In the equations,  $u_{*b}$  is the wall shear (friction) velocity<sup>1</sup>,  $u_b$  the bottom current,  $|\bar{\mathbf{u}}|$  the magnitude of the depth mean current and  $C_{db}$  the bottom drag coefficient given by

$$C_{db} = \left[ \frac{\kappa}{\ln(z_b/z_0)} \right]^2 \quad \text{or} \quad C_{db} = \left[ \frac{\kappa}{\ln(H/z_0) - 1} \right]^2 \quad (7.2)$$

for the 3-D, respectively 2-D case. Here,  $\kappa$  is the Von Karman constant,  $z_b$  height of the first velocity node above the sea bed,  $z_0$  the roughness height and  $H$  the total water depth.

In the discussions of the load formulae, given in the sections below, other formulations, obtained from engineering practice, are mentioned and given here for completeness:

$$C_{db} = \frac{g}{C^2} \quad \text{Chezy} \quad (7.3)$$

$$C_{db} = \frac{gM^2}{H^{1/3}} \quad \text{Manning} \quad (7.4)$$

$$C_{db} = \frac{g}{[18 \log(2H/5z_0)]^2} \quad \text{White-Colebrook} \quad (7.5)$$

The coefficients  $C$  and  $M$  are denoted as the Chezy, respectively Manning coefficients.

In case of a rough bed, the roughness length  $z_0$  can be calculated using the following relation:

$$z_0 = 0.11 \frac{\nu}{u_*} + \frac{k_b}{30} \approx \frac{k_b}{30} \quad (7.6)$$

Here,  $k_b$  is the roughness height from Nikuradse,  $\nu$  the kinematic viscosity and  $u_*$  the friction velocity.

Different kinds of roughness height are considered:

- Physical or “form” roughness  $k_b$  representing the heights of the elements composing the bottom roughness. The corresponding physical bottom stress is used as boundary condition for the momentum flux at the bottom.
- “Skin” roughness  $k_s$  usually related to the median particle size. Skin stress is used in the sediment module to calculate the amount of sediment material resuspended from the sea bed, once the stress exceeds a critical value (see below).

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<sup>1</sup>Since the friction velocity always refers to the bottom in this chapter, the subscript  $b$  will be omitted in the following.

- The main effect of surface waves on the bottom stress is a significant increase of both the form and skin roughness heights. The new roughness height, denoted as  $k_a$ , is obtained from formulae for current-wave interaction theories. The formulations are currently not implemented in COHERENS, but foreseen in a future version of the code.

COHERENS provides different options for the selection of the skin bottom roughness. Either the form roughness, as used by the hydrodynamics, is taken as the skin roughness, or a constant user-defined value is supplied or spatially non-uniform values are provided by the user. A further advantage is that the user can calibrate the sediment transport rates in this way without changing the hydrodynamics. Note that, in the current version of the code, no formulation has been implemented for the skin roughness as function of median size, although  $k_s$  can always be externally supplied by the user.

### 7.2.2 Wave effects

Although surface waves effects have not been implemented in the physical part of the code, the bottom stresses used in the load formulae, discussed below, take account of both currents and waves. A summary of the applied methods is given in this subsection.

The wave field is supplied externally by a significant wave height  $H_s$ , a wave period  $T_w$  and a wave direction  $\phi_w$ . The wave amplitude  $A_w$  and frequency are defined by

$$A_w = H_s/2, \quad \omega_w = 2\pi/T_w \quad (7.7)$$

The near-bottom wave orbital velocity determined from linear wave theory is given by

$$U_w = \frac{A_w \omega_w}{\sinh k_w H} \quad (7.8)$$

with the wavenumber  $k_w$  obtained from the dispersion relation

$$\omega_w^2 = g k_w \tanh k_w H \quad (7.9)$$

The latter equation is solved for  $k_w$  using the approximate formula of Hunt (1979)

$$k_w = \omega_w \sqrt{\frac{f(\alpha)}{gH}} \quad (7.10)$$

with

$$f(\alpha) = \alpha + (1.0 + 0.652\alpha + 0.466422\alpha^2 + 0.0864\alpha^4 + 0.0675\alpha^5)^{-1} \quad (7.11)$$

and

$$\alpha = \frac{\omega_w^2 H}{g} \quad (7.12)$$

With this option, it is assumed that the near bed wave period is equal to the mean wave period, such that the near bed wave excursion amplitude can be written as

$$A_b = \frac{U_w}{\omega_w} \quad (7.13)$$

The combined effect of currents and waves on the bed shear stress is calculated using the method described in Soulsby (1997). Two types of bottom stress are defined

- The mean value over the wave cycle which should be used for currents

$$\tau_m = \tau_c \left[ 1 + 1.2 \left( \frac{\tau_w}{\tau_c + \tau_w} \right)^{3.2} \right] \quad (7.14)$$

where  $\tau_c$  is the bottom stress for currents alone as given by (7.1)–(7.2).

- The maximum value during the wave cycle used in the criterion for resuspension

$$\tau_{max} = \sqrt{\tau_m^2 + \tau_w^2 + 2\tau_m\tau_w \cos \phi_w} \quad (7.15)$$

where the wave bottom stress is defined by

$$\tau_w = \frac{1}{2} f_w U_w^2 \quad (7.16)$$

Two formulations are used for the wave friction

- Swart (1974)

$$\begin{aligned} f_w &= 0.3 \quad \text{for } A_b/k_b \leq 1.57 \\ f_w &= 0.00251 \exp\left(5.21(A_b/k_b)^{-0.19}\right) \quad \text{for } A_b/k_b > 1.57 \end{aligned} \quad (7.17)$$

- Soulsby *et al.* (1993)

$$f_w = 0.237(A_b/k_b)^{-0.52} \quad (7.18)$$

### 7.2.3 Density effects

#### 7.2.3.1 Equation of state

The fluid density is an important parameter for calculating settling velocities and entrainment rates in sediment transport. The equation of state that is used to calculate the fluid density as function of the water temperature and salinity is discussed in Section 4.2.3. When sediment is present, an additional effect is present in the equation of state

$$\rho = \left(1 - \sum_{n=1}^N c_n\right)\rho_w + \sum_{n=1}^N c_n\rho_{s,n} \quad (7.19)$$

Here  $\rho$  is the density of the mixture,  $\rho_w$  the density of the fluid (including the effects of temperature, pressure and salinity),  $c_n$  the volume concentration of fraction  $n$ ,  $N$  the number of sediment fractions and  $\rho_{s,n}$  the particle density for sediment fraction  $n$ . A stable vertical sediment stratification leads to damping of turbulence and affects the settling velocity of the sediment. However, care must be taken when applying this in combination with hindered settling (Section 7.3.3.2), because hindered settling models already account for the increased buoyancy of the mixture. Thus using a changed equation of state in combination with hindered settling will result in too low settling velocities.

#### 7.2.3.2 Density stratification

To a large extent, the two-way coupling effects between flow and sediment transport are due to density stratification effects. Villaret & Trowbridge (1991) showed that the effects of the stratification from suspended sediment are very similar (even with the same coefficients) to the ones produced by temperature and salinity gradients. This means that the effects of sediment-turbulence interaction in COHERENS can be implemented within the existing turbulence models in the same way as  $T$  and  $S$ .

The (squared) buoyancy frequency in the presence of vertical sediment stratification becomes:

$$N^2 = N_w^2 - g \sum_{n=1}^N \beta_{c,n} \frac{\partial c_n}{\partial z} \quad (7.20)$$

where  $N_w^2$  is the value in the absence of sediment stratification, as given by (4.130) and  $\beta_{c,n}$ , the expansion coefficient for sediment fraction  $n$ , which is

calculated from the density of each sediment fraction  $\rho_{s,n}$  using:

$$\beta_{c,n} = \frac{1}{\rho} \frac{\partial \rho}{\partial c_n} = \frac{\rho_{s,n} - \rho_w}{\rho} \quad (7.21)$$

The baroclinic component of the horizontal pressure gradient contains an additional term due to horizontal sediment stratification. Equation (5.223) now becomes

$$-\frac{\partial q}{\partial x_i} \simeq g \int_z^\zeta \left( \beta_T \frac{\partial T}{\partial x_i} - \beta_S \frac{\partial S}{\partial x_i} - \sum_{n=1}^N \beta_{c,n} \frac{\partial c}{\partial x_i} \right) dz' \quad (7.22)$$

In transformed coordinates the following term is added to right hand side of (5.224) for each fraction  $n$ :

$$F_i^{c,n} = -g \int_z^\zeta \beta_{c,n} \left( \frac{\partial c_n}{\partial x_i} \Big|_s - \frac{\partial c_n}{\partial z'} \frac{\partial z'}{\partial x_i} \Big|_s \right) dz' \quad (7.23)$$

The numerical methods, described in Section 5.3.13 for discretising the baroclinic gradient are easily extended to include sediment stratification.

## 7.2.4 Kinematic viscosity

The kinematic viscosity  $\nu$  is an important parameter that has a significant influence on the settling velocity and the critical bed stress. Its value can be selected in COHERENS either as a user-defined constant or as a temperature dependent value using the ITTC (1978) equation for sea water

$$\nu = 10^{-6} [1.7688 + (0.659 \cdot 10^{-3} (T - 1) - 0.05076) (T - 1)] \quad (7.24)$$

Here  $T$  is the water temperature in  $^{\circ}\text{C}$  and  $\nu$  is given in  $\text{m}^2/\text{s}$ .

### Implementation

The following switches are available:

`iopt_sed_tau` Selects type of roughness height  $z_s$  used for sediment transport

- 1: set equal to the form roughness used in the hydrodynamics
- 2: user-defined uniform value
- 3: spatially non-uniform value supplied by the user

`iopt_waves` Disables/enables wave effects and selects type of input data wave data, for use in the sediment transport models.

0: wave effects disabled

1: waves enabled with input of wave height, period and direction

2: waves enabled with input of wave height, period, velocity, excursion and direction

`iopt_sed_dens` Disables (0) or enables (1) effects of sediments in the equation of state and density stratification.

`iopt_kinvisc` Selects type of kinematic viscosity.

0: user-selected uniform value

1: from the ITTC (1978) equation

## 7.3 Sediment properties

### 7.3.1 Introduction

Sediment concentrations can be represented either as a volumetric concentration  $c$  in units of  $\text{m}^3/\text{m}^3$  or as a mass concentration  $c_{mass}$  in  $\text{kg}/\text{m}^3$ . The two forms are related by

$$c_{mass} = \rho_s c \quad (7.25)$$

The volumetric form is taken in COHERENS, which is considered as more physically meaningful, especially in processes as hindered settling.

A dimensional analysis by Yalin (1977) shows that the sediment transport can be expressed by a number of dimensionless parameters:

$$Re_* = \frac{u_* d}{\nu} \quad (7.26)$$

$$\theta = \frac{\rho u_*^2}{(\rho_s - \rho)gd} \quad (7.27)$$

$$s = \frac{\rho_s}{\rho} \quad (7.28)$$

$$d_* = d \left[ (s - 1) \frac{g}{\nu^2} \right]^{1/3} \quad (7.29)$$

$$\Phi = \frac{q}{\sqrt{(s - 1)gd^3}} \quad (7.30)$$

where  $d$  is the particle diameter,  $Re_*$  the particle Reynolds number,  $\theta$  the dimensionless shear stress or Shields parameter (Shields, 1936),  $s$  the relative density,  $d_*$  the dimensionless particle diameter and  $q$  the sediment load per unit width (in  $\text{m}^2/\text{s}$ ).

These non-dimensional diameters are often used to describe sediment properties, such as critical shear stresses or settling velocities, which are discussed below.

### 7.3.2 Critical shear stress

In many transport relations, a critical shear stress is required, which is the value of the bed shear stress at which the sediment particles start to move (the threshold of motion). The value of the bed shear stress is in engineering practice obtained from the Shields curve, which relates the non-dimensional critical shear stress  $\theta_{cr}$  to the particle Reynolds number  $Re_*$ .

For numerical models, some fits to these curves are made, expressing  $\theta_{cr}$  as a function of  $d_*$  rather than  $Re_*$ , in order to avoid the iteration process necessary for the determination of the threshold of motion in the original curve obtained by Shields (1936), as both  $\theta$  and  $Re_*$  are a function of  $u_*$ . Brownlie (1981) obtained the following relation from a fit through the data:

$$\theta_{cr} = 0.22d_*^{-0.9} + 0.06.10^{(-7.7d_*^{-0.9})} \quad (7.31)$$

An alternative form, also obtained by fitting of the Shields curve, is proposed by Soulsby & Whitehouse (1997)

$$\theta_{cr} = \frac{0.3}{1 + 1.2d_*} + 0.055 (1 - e^{-0.02d_*}) \quad (7.32)$$

Another equation, available in COHERENS from Wu *et al.* (2000), is to assume a constant critical Shields stress

$$\theta_{cr} = 0.03 \quad (7.33)$$

This equation should only be used in combination with the bed load and total load equations from Wu *et al.* (2000). In COHERENS, the critical shear stress is calculated for each fraction separately at each horizontal location, using the local values of the kinematic viscosity and the mixture density.

The sediment transport model in COHERENS has an option available for the user to manually set the critical shear stress. The user can either set the critical shear stress to a uniform value throughout the whole domain, or to a spatially varying value. Be aware that the value given should be the kinematic critical shear stress defined as the square of the critical shear velocity  $u_{*,cr}^2 = \tau_{cr}/\rho$ .



### 7.3.2.1 Hiding and exposure

A hiding and exposure factor can be implemented for the critical shear stress  $\tau_{cr}$  or the critical Shields parameter  $\theta_{cr}$  to account for the change in sediment transport when different fractions are present. In general, the correction factor will increase the critical value for the smaller, hidden fractions and will reduce the critical threshold for motion for the coarser, exposed, fractions. Several formulations can be found in literature, usually as a function  $\xi(d_n/d_{50})$ .

Two methods are available in COHERENS for hiding and exposure. The first one uses the formulation of Wu *et al.* (2000) based upon a stochastic relation between size and gradation of bed materials and the hidden and exposed probabilities. The probability of particles  $d_m$  in front of particles  $d_n$  can be assumed to be the fraction of particles  $d_m$  in the bed material,  $p_{bm}$ . The total hidden and exposed probabilities of particles  $d_n$  can be described by:

$$p_{hn} = \sum_{m=1}^N p_{bm} \frac{d_m}{d_n + d_m} \quad (7.34)$$

$$p_{en} = \sum_{m=1}^N p_{bm} \frac{d_n}{d_n + d_m} \quad (7.35)$$

and  $p_{hn} + p_{en} = 1$ . The hiding/exposure factor is then defined by:

$$\xi_n = \left( \frac{p_{en}}{p_{hn}} \right)^m = \left( \frac{1 - p_{hn}}{p_{hn}} \right)^m \quad (7.36)$$

where  $m$  is an empirical parameter which Wu *et al.* (2000) determined as  $m = -0.6$  in their study.

Alternatively, hiding and exposure can be calculated with the equation of Ashida & Michiue (1972), which is an adapted form of Egiazaroff (1965)

$$\xi_n = \begin{cases} 0.8429 \frac{d_{50}}{d_n} & \text{if } \frac{d_{50}}{d_n} < 0.38889 \\ \left[ \frac{\log 19}{\log 19 + \log(d_n/d_{50})} \right]^2 & \text{if } \frac{d_{50}}{d_n} \geq 0.38889 \end{cases} \quad (7.37)$$

where  $d_{50}$  is the median grain size diameter, which, in case multiple size fractions are present, means that particles with sizes less than  $d_{50}$  account for 50% of the total mass.

### 7.3.2.2 Bed level gradient

The threshold of motion is also influenced by the bed level gradient in the current direction

$$\theta_{cr} = \theta_{cr,0} \left( 1 - \frac{\partial h}{\partial c} \right) \quad (7.38)$$

where  $\theta_{cr,0}$  is the critical Shields parameter in absence of bed slopes,  $h$  the mean water depth and  $\partial h/\partial c$  the bed slope along the current direction, defined below (equation (7.81)). Since many of the bed boundary conditions for suspended sediment concentrations depend on the critical Shields parameters, this correction will also affect the amount of sediment in suspension.

## 7.3.3 Settling velocity

### 7.3.3.1 Single particle settling

Different methods are available to calculate the settling velocity of sediment particles in COHERENS. A general formula for settling velocity was derived by Camenen (2007), based on two formulations for the drag coefficient:

$$w_s = \frac{\nu}{d} Re_p = \frac{\nu}{d} \left[ \sqrt{\frac{1}{4} \left( \frac{A}{B} \right)^{2/m} + \left( \frac{4 d_*^3}{3 B} \right)^{1/m}} - \frac{1}{2} \left( \frac{A}{B} \right)^{1/m} \right]^m \quad (7.39)$$

with  $d_*$  defined by (7.29) and  $Re_p = w_s d_p / \nu$  the particle Reynolds number.

The coefficients  $A$ ,  $B$  and  $m$  have been given different values in literature. Camenen (2007) recalibrated these coefficients using results of Dietrich (1982) and data from 11 previous studies. He found for natural sand that  $A = 24.6$ ,  $B = 0.96$  and  $m = 1.53$ , for flocs he found  $A = 26.8$ ,  $B = 2.11$  and  $m = 1.19$ . Note that when using this equations for calculating the settling velocity for mud flocs, the diameter that is used is the floc diameter, not the one of the primary particles.

For small Reynolds numbers ( $Re_p \ll 1$ ), the settling velocity can be calculated from the Stokes equation (Stokes, 1847), valid for small spherical particles

$$w_s = \frac{sgd^2}{18\nu} \quad (7.40)$$

where  $s$  is the relative density defined by (7.28),  $g$  the acceleration due to gravity,  $d$  the particle diameter and  $\nu$  the viscosity. This equation should only be used for very small particles, because it can give large overestimations for larger ones.

The following formulation is recommended by Soulsby (1997) for natural sands

$$w_s = \frac{\nu}{d} \left[ \sqrt{10.36^2 + 1.049d_*^3} - 10.36 \right] \quad (7.41)$$

Another equation for the settling velocity, available in COHERENS is the one by Zhang & Xie (1993)

$$w_s = \sqrt{\left(13.95 \frac{\nu}{d}\right)^2 + 1.09(s-1)gd} - 13.95 \frac{\nu}{d} \quad (7.42)$$

### 7.3.3.2 Hindered settling

In case of high concentrations (mass concentrations larger than 3 g/l) the settling of particles is not only influenced by the surrounding fluid and the weight and shape of the particles, but also by the other particles in suspension. The settling of the particles is reduced by a number of processes, e.g. return flow, particle collisions, changed mixture viscosity, buoyancy due to increased mixture density and wake formation.

For sand, hindered settling can be calculated with the equation of Richardson & Zaki (1954). They described the settling velocity in high sediment concentrations as a function of the sediment concentration,  $Re_p$  and undisturbed settling velocity  $w_{s,0}$  (i.e. the one of a single particle without being disturbed):

$$w_s = w_{s,0} (1 - c)^n \quad (7.43)$$

In this equation,  $n$  is a user-defined constant. Note that this formula is not advisable for cohesive sediments because it does not take into account the maximum sediment concentration (i.e. the settling velocity should reduce to zero at the maximum packing concentration  $c_{max}$ ). This is not a severe disadvantage, because in most practical applications, the concentrations are not so high, except close to the sea bed. Note that some hindered settling effects are already calculated when the fluid density depends on the sediment concentrations, and that this equations are only valid when a net deposition occurs (Breugem, 2012). Therefore, it is recommended not to use hindered settling in that case.

Winterwerp & van Kesteren (2004) introduced a different formula for mud

$$w_s = w_{s,0} \frac{(1 - c_f)(1 - c)}{1 + 2.5c_f} \quad (7.44)$$

Here  $w_{s,0}$  is the settling velocity of a single mud-floc,  $c_f = c/c_{gel}$ ,  $c_{gel}$  is the gelling concentration (the mass concentration at which the mud flocs form a

space filling network), which can be calculated with

$$c_{gel} = \rho_s \frac{d_p^{3-n_f}}{d_f} \quad (7.45)$$

The parameters needed to calculate the gelling concentration ( $n_f$  and  $d_f$ ) are difficult to determine. Therefore, the user must provide a value for  $c_{gel}$ , which should be obtained from measurements.

### 7.3.3.3 Influence of flocculation

In COHERENS, the influence of turbulence and sediment concentration on the flocculation of mud flocs can be simulated. The effect of salinity is not considered. For simulating the influence of turbulence the empirical model by Van Leussen (1994) is used for the fall velocity of settling flocs

$$\frac{w_s}{w_{s,r}} = \frac{1 + aG}{1 + bG^2} \quad (7.46)$$

Here  $w_{s,r}$  is a reference velocity,  $G$  the shear rate, defined as  $G = \sqrt{\varepsilon/\nu}$  and  $a$ ,  $b$  empirical constants. In order to estimate the shear rate if the turbulent dissipation  $\varepsilon$  is not known (e.g. when an algebraic turbulence model is used),  $G$  can be obtained assuming equilibrium between production and dissipation of turbulent kinetic energy (4.204) which gives <sup>2</sup>

$$G = \sqrt{\frac{\nu_T M^2 - \lambda_T N^2}{\nu}} \quad (7.47)$$

where  $\nu_T$ ,  $\lambda_T$  are the turbulent diffusion coefficients for respectively momentum and density and  $M$ ,  $N$  the shear and buoyancy frequencies (see Section 4.4 for details).

A similar empirical approach to flocculation, which considers the influence of the sediment concentration is given in Van Rijn (2007b)

$$\frac{w_s}{w_{s,r}} = [4 + \log_{10}(2c/c_{gel})]^\alpha = \phi_{floc} \quad (7.48)$$

with  $1 \leq \phi_{floc} \leq 10$  and  $\alpha$  a user-defined constant with a minimum of 0 and a maximum of 3, whose default value in COHERENS was obtained from calibration of flocculation data in the Scheldt and harbour of Zeebrugge. Note

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<sup>2</sup>Note that  $G$  has a different meaning than in Section 4.4 where it represents the buoyancy term in the turbulent energy equation.

that Van Rijn (2007b) suggests  $\alpha = d_{sand}/d_{50} - 1$ , with  $d_{sand}$  the diameter of the transition from sand to silt ( $62 \mu\text{m}$ ).

It is also possible to combine the effects of these two models in COHERENS. In this case, the settling velocity is multiplied by the two corrections factors on the right of (7.46) and (7.48).

### Implementation

The methods in this section are selected with the following switches

`iopt_sed_taucr` Selects type of method for the critical shear stress

- 1: user-defined value for each fraction
- 2: Brownlie (1981) equation (7.31)
- 3: Soulsby & Whitehouse (1997) equation (7.32)
- 4: Wu *et al.* (2000) equation (7.33)

`iopt_sed_hiding` Type of hiding/exposure factor for the critical shear stress

- 0: hiding disabled
- 1: Wu *et al.* (2000) equation (7.36)
- 2: Ashida & Michiue (1972) equation (7.37)

`iopt_sed_ws` Type of method for the settling velocity

- 1: user-defined value for each fraction
- 2: Camenen (2007) formulation (7.39) for sand
- 3: Camenen (2007) formulation (7.39) for mud
- 4: Stokes formula (7.40)
- 5: Soulsby (1997) formula (7.41)
- 6: Zhang & Xie (1993) equation (7.42)

`iopt_sed_hindset` Formulation for hindered settling

- 0: hindered settling disabled
- 1: Richardson & Zaki (1954) equation (7.43)
- 2: Winterwerp & van Kesteren (2004) formula (7.44)

`iopt_sed_floc` Type of flocculation factor for the settling velocity

- 0: flocculation effect disabled
- 1: Van Leussen (1994) equation (7.46)
- 2: Van Rijn (2007b) equation (7.48)
- 3: combination of the two previous methods

## 7.4 Bed load

### 7.4.1 Introduction

In this section an overview of available models for bed load transport is given. These models are applicable to non-cohesive sediment transport and can be used in combination with suspended transport equations except for the total load models, discussed in Section 7.5 below. Bed load is active in the near-bed layer with thickness taken often as  $z_{sb} \approx 2d$ . Both states are coupled by the reference concentration  $c_a$ , which is defined as the sediment concentration in the near-bed layer.

Bed load is usually expressed as the *intensity of solid discharge*, which can be written in non-dimensional form (see also equation (7.30)):

$$\Phi_b = \frac{q_b}{\sqrt{(s-1)gd^3}} \quad (7.49)$$

where  $q_b$  is the bed load solid discharge per unit width in  $\text{m}^2/\text{s}$ .

The Meyer-Peter-Muller and the Einstein equations are two examples using dimensionless parameters for relating solid discharge to the shear stress at the bed. However, the solid discharge rates of bed load, suspended load and total load in this numerical model are given as the dimensional quantities expressing the volumetric discharge per unit flow-perpendicular width (in  $\text{m}^2/\text{s}$ ).

The bed load transport is assumed to occur below the roughness level  $z_0$ , i.e. below the lowest  $\sigma$ -layer. The bed load transport vector is decomposed in the same manner as the horizontal velocity vector. The bed load transport magnitude is computed as the vector sum of transport components along the  $\xi_1$  and  $\xi_2$  directions, assuming local orthogonality of the computational grid:

$$\mathbf{q}_b = (q_{b1}, q_{b2}) \quad (7.50)$$

The  $\xi_1$  and  $\xi_2$  components are computed at the U-, respectively V-nodes grid points. This avoids that interpolations of the velocity components to e.g. the C-points need to be taken prior to the determination of sediment transport rates.

#### 7.4.1.1 Transport of different sediment size classes

COHERENS permits to simulate the transport of different sediment classes ( $n = 1, \dots, n_f$ ), each having a fraction  $f_n$  present in the bed sediments. Each sediment class will experience hiding or exposure due to the presence of grains of different size in its vicinity. The effect is introduced in the model

by means of a hiding/exposure factor (see Section 7.3.2.1 for more detailed information).

#### 7.4.1.2 Applicability of transport models

Every sediment transport formulation has been developed and calibrated under controlled conditions where a certain range of parameters such as particle diameters has been covered. The formulations are therefore only valid under the conditions covered by the experiments that lead to the formula with tuned parameters. The model user should take these limitations into account when applying a formula. The range of applicability of a number of transport formula has been given in table 7.1.

Table 7.1: Overview of applicability of sediment transport formula, i.e. conditions of the measurements used as calibration data (if available).

	d [mm]	$d_x$ , for non-uniform granulate
Meyer-Peter & Müller (1948)	3.1-28.6	$d_{50}$
Ackers & White (1973) <sup>a</sup>	0.04-4.0	$d_{35}$
Engelund & Fredsøe (1976)	0.19-0.93	$d_{50}$
Van Rijn (1993)	0.2-2.0	$d_{50}$
Wu <i>et al.</i> (2000)	0.06-32	graded

<sup>a</sup>Discussed in Section 7.5 below

#### 7.4.2 Meyer-Peter and Mueller (1948)

The formula of Meyer-Peter & Müller (1948) relates the Shields parameter  $\theta_n$  to the intensity of solid discharge  $\Phi_n$  for sediment class  $n$ , with the following formula

$$\Phi_n = 8f_n(\xi_M\theta_n - \xi_n\theta_{cr,n})^{3/2} \quad (7.51)$$

Here,  $\theta_{cr,n}$  is the critical dimensionless shear stress, the threshold above which motion of the sediment particles begins, and  $\xi_n$  the hiding or exposure factor for sediment class  $n$ . The parameter  $\xi_M$  is a roughness parameter expressing the relative importance of bed forms in the total roughness, equal to 1 in absence of bed forms and  $1 > \xi_M > 0.35$  if bed forms are present:

$$\xi_M = \left(\frac{M'}{M}\right)^{3/2} \quad (7.52)$$

where  $M$  is Manning's roughness parameter for the total roughness and  $M'$  the Manning roughness for granulates only [ $\text{m}^{-1/3}\text{s}$ ]. In the current version of the model, separate bed form roughness has not yet been implemented, so that the factor  $\xi_M$  is always set to 1. However, when the user specifies the friction factor to be used for sediment transport calculations as the skin friction roughness, an equivalent result is obtained as bed load transport is assumed to be independent of form drag in this model (see Section 7.2.1).

It is important to mention under which circumstances a sediment transport relation has been developed, and hence its validity range. The Meyer-Peter-Muller formula (e.g.) is applicable for large sediment grain sizes, i.e.  $d > 3.1$  mm. See also table 7.1.

### 7.4.3 Engelund and Fredsøe (1976)

From an equilibrium of agitating and stabilizing forces, a dimensionless expression was found for the bed load transport. The agitating forces consist of drag and lift, while the stabilising forces are reduced gravity and friction. The agitating forces can be expressed as a drag force:

$$F_D = \frac{c_l}{2} \rho U_p^2 \frac{\pi d^2}{4} \quad (7.53)$$

The coefficient  $c_l$  can be seen as a correction of the drag force for lifting,  $U_p$  is the particle migration velocity.

The friction force experienced by the particles can be expressed as:

$$F_f = \rho g (s - 1) \frac{\pi d^3}{6} \beta \quad (7.54)$$

where  $\beta$  is a dynamic friction coefficient.

When both forces are in equilibrium the following is obtained:

$$\frac{U_p}{u_*} = \alpha \left[ 1 - \sqrt{\theta_0/\theta} \right] \quad (7.55)$$

in which  $\theta_0 = 4\beta/3\alpha^2c$  and  $\alpha = 9$  represents the ratio of the current, taken at two grain sizes from to the bottom, to the friction velocity.

Using measurements to determine the parameters, and using the number of particles in the bed load layer per unit area, based on a particle probability  $p$ , the following bed load transport equation is found:

$$q_b = 9.3 \frac{\pi}{6} d p u_* \left[ 1 - 0.7 \sqrt{\theta_{cr}/\theta} \right] \quad (7.56)$$



or, in non-dimensional form:

$$\Phi = 5p(\sqrt{\theta} - 0.7\sqrt{\theta_{cr}}) \quad (7.57)$$

In the case of fractional transport this becomes:

$$\Phi_n = 5pf_n(\sqrt{\theta} - 0.7\sqrt{\xi_n\theta_{cr,n}}) \quad (7.58)$$

The probability of mobility of a sediment grain  $p$  is expressed as follows:

$$p = \left[ 1 + \left( \frac{\pi f_d / 6}{\theta - \theta_{cr}} \right)^4 \right]^{-1/4} \quad (7.59)$$

with  $\theta > \theta_{cr}$  and  $f_d$  the dynamic friction coefficient of a grain, assumed to be equal to 0.51.

This relation was proven to be accurate for fine to medium sands, with experiments using sand with diameters of 190, 270 and 930  $\mu\text{m}$ .

#### 7.4.4 Van Rijn (1984b)

Van Rijn (1984b) used the method of Bagnold (1954) to determine the bed load sediment transport. Bed load transport is here dominated by particle saltation with height  $\delta_b$  causing a bed load concentration  $c_b$  with particles moving at a velocity equal to  $U_p$ . Unlike in the formulation of Engelund & Fredsøe (1976), only drag and lift forces as well as reduced weight are taken into account. The bed load transport was then assumed to be  $q_b = U_p \delta_b c_b$ .

The bed load transport by currents for particles with diameter in the range of 200 to 2000  $\mu\text{m}$  is determined by:

$$\Phi_n = 0.053 f_n T_n^{2.1} d_*^{-0.3} \quad (7.60)$$

where

$$T_n = \max \left( \frac{\tau_b}{\tau_{cr,n}} - 1, 0 \right) \quad (7.61)$$

The implementation of this formula also allows for the computation of transport of different size classes  $n$  with different bed fractions  $f_n$ .

#### 7.4.5 Wu et al. (2000)

This equation is specially recommended for the calculation of graded sediment, because in its determination, the different fractions were explicitly taken into account.

The following steps are taken to calculate the suspended load and bed load transport. Firstly, the settling velocity of fraction  $n$  is calculated with formula (7.42) from Zhang & Xie (1993). Then the hiding and exposure factors  $p_{hn}$  and  $p_{en}$  are computed as defined in Section 7.3.2.1.

The critical shear stress (taking into account hiding and exposure) for fraction  $n$  is calculated:

$$\tau_{cr,n} = 0.03(\rho_s - \rho)gd_n \left( \frac{p_{hn}}{p_{en}} \right)^{0.6} \quad (7.62)$$

The bed load sediment transport is then determined with:

$$\Phi_{b,n} = 0.0053f_n \left[ \max \left( \frac{\tau_b}{\tau_{cr,n}} - 1, 0 \right) \right]^{2.2} \quad (7.63)$$

whereby the hiding/exposure factor  $\xi_n$  for sediment class  $n$ , is always calculated with the method of Wu *et al.* (2000), as given by (7.34)–(7.36).

#### 7.4.6 Soulsby (1997)

The bed load transport equation by Soulsby (1997) is an approximation obtained by integrating the bed load equation over one wave period with a sinusoidal oscillatory component. The components  $\Phi_{bc}$  and  $\Phi_{bn}$  along the directions parallel, respectively perpendicular to the current direction are obtained as follows

$$\begin{aligned} \Phi_{bc,1} &= 12\sqrt{\theta_m}(\theta_m - \theta_{cr,n}) \\ \Phi_{bc,2} &= 12(0.95 + 0.19 \cos 2\phi_w)\sqrt{\theta_w\theta_m} \\ \Phi_{bc} &= f_n \max[\Phi_{bc,1}, \Phi_{bc,2}] \\ \Phi_{bn} &= f_n \frac{12(0.19\theta_m\theta_w^2 \sin 2\phi_w)}{\theta_w^{1.5} + 1.5\theta_m^{1.5}} \end{aligned} \quad (7.64)$$

and  $\Phi_{bc} = \Phi_{bn} = 0$  if  $\theta_{max} \leq \theta_{cr}$ .  $\theta_m$  is the value of  $\theta$  averaged over the wave cycle,  $\theta_w$ , is the amplitude of the oscillatory component of  $\theta$  and  $\phi$  the angle between current and wave propagation direction. The maximal value of the Shields parameter  $\theta_{max}$  over the wave cycle is calculated as follows

$$\theta_{max} = \sqrt{\theta_m^2 + \theta_w^2 + 2\theta_m\theta_w \cos^2 \phi_w} \quad (7.65)$$

The wave-averaged Shields parameter  $\theta$  is determined by the relationship from Soulsby (1995):

$$\theta_m = \theta_c \left[ 1 + 1.2 \left( \frac{\theta_w}{\theta_c + \theta_w} \right)^{3.2} \right] \quad (7.66)$$

where  $\theta_c$  is the current related Shields parameter. Note that (7.66) and (7.65) are the dimensionless versions of (7.14)–(7.15).

From the equations it can be seen that when the wave direction is perpendicular or along the current direction, the component at right angle to the current direction is zero. No wave asymmetry is taken into account.

In the absence of waves, one has  $\Phi_{bc,2} = \Phi_{bn} = 0$  so that

$$\Phi_b = \Phi_c = 12\sqrt{\theta} \max(\theta - \theta_{cr,n}, 0) \quad (7.67)$$

The formula is implemented on the numerical grid by computing  $\Phi_{bc}$  and  $\Phi_{bn}$  both at U- and V-velocity points, after which they are projected on the  $\xi_1$  and  $\xi_2$  grid orientation. More details are given in Section 7.7.2.

#### 7.4.7 Van Rijn (2007a)

In an unpublished paper by Van Rijn (2003) an approximative bed load transport formula was determined in combination with time-integrated formulations for wave-related and current-related suspended sediment transport. In this sediment transport code, it was decided to combine the modified full equation (instantaneous transport integrated over a wave period) by Van Rijn (1993) with the approximative formulations for suspended load (see Section 7.5), resulting in the total sediment transport. The instantaneous bed load formula is the one given in Van Rijn (1993) but with modifications on a calibration factor and the exponent  $\eta$

$$q_b = f_n \gamma \rho_s d_{50} d_*^{-0.3} (\tau_{b,cw}/\rho)^{1/2} [(\tau_{b,cw} - \tau_{b,cr})/\tau_{b,cr}]^\eta \quad (7.68)$$

in which

$$\tau_{b,cw} = 0.5 \rho_w f_{cw} (U_{\delta,cw})^2 \quad (7.69)$$

$$f_{cw} = \alpha \beta f_c + (1 - \alpha) f_w \quad (7.70)$$

$$f_c = 2 \left[ \frac{\kappa}{\ln(15H/k_s)} \right]^2 \quad (7.71)$$

$$f_w = 0.00251 \exp\left(5.21(A_w/k_s)^{-0.19}\right) \quad (7.72)$$

and  $f_w$  given by (7.18).

$f_n$	fraction in the bed of sediment class $n$
$\tau_{b,cw}$	instantaneous grain-related bed-shear stress due to both currents and waves
$U_{\delta,cw}$	instantaneous velocity magnitude due to currents and waves at the edge of the wave boundary layer
$f_{cw}$	skin friction coefficient due to current and waves
$f_c$	current-related skin friction coefficient
$f_w$	wave-related skin friction coefficient
$k_s$	skin roughness height = $d_{90} = 1.5d_{50}$
$\alpha$	coefficient related to the relative strength of current and wave motion
$\beta$	coefficient related to the vertical structure of the velocity profile (Van Rijn, 1993)
$A_b$	near bed wave excursion amplitude
$\tau_{b,cr}$	critical bed-shear stress
$f_{silt}$	silt factor: $f_{silt} = \max(1, d_{sand}/d_{50})$
$\gamma$	coefficient = 0.5
$\eta$	exponent = 1.0

The factor  $\beta$  converts the depth-averaged flow dependence of  $f_c$  to a wave boundary layer friction factor. Furthermore, this factor shifts the applied roughness length from the grain roughness  $k_s$  to an apparent roughness length  $k_a$ . The factor  $\beta$  is determined by (Appendix A in Van Rijn, 1993):

$$\beta = 0.25 \left[ \frac{\ln(30h/k_s) - 1}{\ln(30\delta/k_s)} \right]^2 \quad (7.73)$$

where  $\delta = \max(3\delta_w, k_s)$  and  $\delta_w$  the thickness of the wave boundary layer

$$\delta_w = 0.072A_b \left( \frac{A_b}{k_s} \right)^{-0.25} \quad (7.74)$$

with  $A_b$  given by (7.13).

The current ratio factor  $\alpha$  is defined by

$$\alpha = \frac{u_{c\delta}}{u_{c\delta} + U_w} \quad (7.75)$$

The wave-mean current magnitude at the top of the boundary layer is given by

$$u_{c\delta} = \frac{|\bar{\mathbf{u}}_c| \ln(30\delta/k_a)}{\ln(30H/k_a) - 1} \quad (7.76)$$

where  $|\bar{\mathbf{u}}_c|$  is the depth-mean current magnitude and the apparent roughness  $k_a$  represents the influence of waves on the roughness length:

$$k_a = k_s \exp(\gamma U_w / \bar{u}_c) \quad (7.77)$$

$$\gamma = 0.8 + \phi - 0.3\phi^2 \quad (7.78)$$

where  $\phi$  is the angle between wave propagation and current direction and  $U_w$  the near-bed wave orbital velocity given by (7.8).

The instantaneous near-bed flow velocity magnitude due to combined waves and currents is determined as the magnitude of the vector sum of the current and wave components

$$U_{\delta,cw}(t) = \left[ (U_w \sin \varphi)^2 + u_{c\delta}^2 + 2u_{c\delta}U_w \cos \phi \sin \varphi \right]^{1/2} \quad (7.79)$$

where  $\varphi = \omega_w t$  is the wave phase.

The instantaneous bed load transport vector is decomposed in a component along the current direction and a component at right angle to the current. The mean components are obtained after integration over the wave period, by means of the Gaussian quadrature technique (Section 7.7.3).

$$\begin{aligned} \bar{q}_{bc} &= \frac{1}{2\pi} \int_0^{2\pi} q_b(t) \frac{u_{c,\delta} + U_w \sin \varphi \cos \phi}{U_{\delta,cw}(t)} d\varphi \\ \bar{q}_{bn} &= \frac{1}{2\pi} \int_0^{2\pi} q_b(t) \frac{U_w \sin \varphi \sin \phi}{U_{\delta,cw}(t)} d\varphi \end{aligned} \quad (7.80)$$

Note that the transport components are calculated at U- and V-nodes separately.

#### 7.4.8 Van Rijn (2003)

In the current version the bed load transport equations are identical to the Van Rijn (2007a) method when this formula is selected. The difference between both methods is situated in the suspended load equations (see Sections 7.5.5 and 7.5.6).

#### 7.4.9 Bed slope effects and coordinate transforms

The gradient of the bed level can have three effects:

- influence of local flow velocity
- a different critical condition for incipient motion
- a change in the bed load transport rate and direction beyond incipient motion.

In COHERENS, the second and third effect can be modeled using the switch `iopt_sed_slope`. The second effect was discussed in Section 7.3.2. For the third effect, a correction for the transport rate vector magnitude has to be combined with a correction on the transport direction, i.e. the correction has a longitudinal component parallel to the current vector and transverse component, normal to the current.

The longitudinal  $\partial h/\partial c$  and transverse  $\partial h/\partial n$  bed slopes are given by

$$\frac{\partial h}{\partial c} = \frac{1}{h_1} \frac{\partial h}{\partial \xi_1} \cos \phi_c + \frac{1}{h_2} \frac{\partial h}{\partial \xi_2} \sin \phi_c \quad (7.81)$$

$$\frac{\partial h}{\partial n} = \frac{1}{h_2} \frac{\partial h}{\partial \xi_2} \cos \phi_c - \frac{1}{h_1} \frac{\partial h}{\partial \xi_1} \sin \phi_c \quad (7.82)$$

where  $\phi_c$  is the angle between the  $\xi_1$ - and current directions.

The slope effects are modeled using the relation of Koch & Flokstra (1981). They developed the following relation for a correction factor  $\alpha_s$  on the bed load transport rate

$$\alpha_s = 1 - \beta_s \frac{\partial h}{\partial c} \quad (7.83)$$

with  $\beta_s = 1.3$ . They included also the effect of the slope transverse to the flow by the change in direction of  $\delta_s$  degrees:

$$\tan \delta_s = \beta_s \frac{\partial h}{\partial n} \quad (7.84)$$

Note that these corrections are only applied for bed load equations and not for total load.

The bed load formulae in the previous subsections were derived along the current direction and an additional transverse component in the presence of waves. The components along the coordinate directions are calculated by the following transformations

$$\begin{aligned} q_{b1} &= \alpha_s \left( q_{bc} \cos(\phi_c + \delta_s) - q_{bn} \sin(\phi_c + \delta_s) \right) \\ q_{b2} &= \alpha_s \left( q_{bc} \sin(\phi_c + \delta_s) + q_{bn} \cos(\phi_c + \delta_s) \right) \end{aligned} \quad (7.85)$$

where

- $\alpha_s = 1$ ,  $\beta_s = 0$ ,  $\delta_s = 0$  in the absence of bed slope effects
- $q_{bn} = 0$  in the absence of waves.

## Implementation

The bed load formulae in this section are selected with the following switches

`iopt_sed_bedeq` Type of bedload equation.

- 1 : Meyer-Peter & Müller (1948)
- 2 : Engelund & Fredsøe (1976)
- 3 : Van Rijn (1984b)
- 4 : Wu *et al.* (2000)
- 5 : Soulsby (1997). This equation includes wave effects.
- 6 : Van Rijn (2003). This formula includes wave effects.
- 7 : Van Rijn (2007a). This method includes wave effects.

`iopt_sed_slope` Disables (0) or enables (1) bed slope effects using the Koch & Flokstra (1981) formulation.

## 7.5 Total Load

### 7.5.1 Engelund and Hansen (1967)

The classic formula of Engelund & Hansen (1967) has been modified by Chollet & Cunge (1979) to take into account of different transport regimes and to introduce a threshold for beginning of sediment particle motion:

$$q_t = 0.05 f_n \left[ \frac{(s-1)d_{50}^3}{g} \right]^{1/2} K^2 H^{1/3} \theta_*^{5/2} \quad (7.86)$$

in which  $f_n$  is the fraction in the bed of sediment class  $n$ ,  $K$  is the Strickler coefficient, i.e. the inverse of the Manning coefficient, defined in (7.4) and  $s$  the relative density of the sediment.

The need for the Strickler coefficient can be avoided by setting  $K^2 H^{1/3}$  equal to  $g/C_D$ , which can be computed using (7.2) with  $z_0$  chosen equal to either the (form) roughness length from the hydrodynamic model or the skin roughness length defined in the sediment transport model. To switch between the two options, set `iopt_sed_tau` to the appropriate value.

In the original formulation by Engelund & Hansen (1967)  $\theta_*$  is set to  $\theta$ . In the modified version of Chollet & Cunge (1979),  $\theta_*$  is defined depending

on the transport regime:

$$\begin{aligned}
 \theta_* &= 0 && \text{if } \theta < 0.06 && \text{(no transport),} \\
 \theta_* &= [2.5(\theta - 0.06)]^{0.5} && \text{if } 0.06 < \theta < 0.384 && \text{(dune regime),} \\
 \theta_* &= 1.065\theta^{0.176} && \text{if } 0.348 < \theta < 1.08 && \text{(transition regime),} \\
 \theta_* &= \theta && \text{if } 1.08 < \theta && \text{(sheet flow regime),}
 \end{aligned} \tag{7.87}$$

where  $\theta$  is the Shields parameter, defined by (7.27).

The user can choose to use the original version as well as the modified version by setting the switch `iopt_sed_eha`. The modified version shows deviations of the transport calculated for bed shear below the sheet flow regime, where the original version shows result more in line with other formulae.

### 7.5.2 Ackers and White (1973)

A direct determination of the total sediment transport rate is provided with the formula from Ackers & White (1973). It gives extra attention to the subdivision of non-cohesive sediment transport for coarser and finer fractions. It was developed using hydraulic considerations and dimensional analysis. The coefficients in the model have been determined by evaluation of nearly 1000 experiments in the laboratory and 250 experiments in the field. Nineteen different transport formulae have been studied in this work.

A parameter of mobility was defined, based on the Shields parameter:

$$F_{gr} = \frac{u_*^{n_w}}{\sqrt{(s-1)gd}} \left[ \frac{|\bar{\mathbf{u}}|}{2.46 \ln(10H/d)} \right]^{(1-n_w)} \tag{7.88}$$

The parameter of transport is then defined as

$$\begin{aligned}
 G_{gr} &= C_w \left( \frac{F_{gr}}{A_w} - 1 \right)^{m_w} && \text{for } F_{gr} > A_w \\
 G_{gr} &= 0 && \text{otherwise}
 \end{aligned} \tag{7.89}$$

The total load is then calculated as follows:

$$(q_{t1}, q_{t2}) = f_n G_{gr} d \left( \frac{|\bar{\mathbf{u}}|}{u_*} \right)^{n_w} (\bar{u}, \bar{v}) = f_n G_{gr} d C_D^{-n_w/2} (\bar{u}, \bar{v}) \tag{7.90}$$

with  $d = d_{35}$ ,  $f_n$  the fraction of size class  $n$  in the bed and  $|\bar{\mathbf{u}}|$  the magnitude of the depth averaged current.

This transport formula was derived by uniting bed load and suspended load transport relations in one function through a transition in the dimensionless  $d_*$  parameter. A revised set of values for the parameters  $m_w$  and



$C_w$  which are used in the current implementation, have been derived after experiments in 1990:

$$\begin{aligned} n_w &= 1 - 0.243 \ln \tilde{d}_* \\ m_w &= 1.67 + 6.83/\tilde{d}_* \\ \ln C_w &= 2.79 \ln \tilde{d}_* - 0.426(\ln \tilde{d}_*)^2 - 7.97 \\ A_w &= 0.14 + 0.23/\tilde{d}_*^{1/2} \end{aligned} \quad (7.91)$$

with  $\tilde{d}_* = \min(60, \max(1, d_*))$ .

Since this total load formula was developed using a  $d_{35}$  grain size diameter, the user should also give this sieve size instead of  $d_{50}$  when the information is available.

### 7.5.3 Madsen and Grant (1976)

The Madsen & Grant (1976) formula is a total load formula determining the instantaneous transport rate vector  $\mathbf{q}_t$ , for one sediment grain size class:

$$\mathbf{q}_t = 40 f_n w_s d_n \theta_{cw}^3 \frac{\mathbf{U}_{cw}(t)}{|\mathbf{U}_{cw}(t)|} \quad (7.92)$$

in which  $w_s$  is the particle fall velocity,  $d_n$  the grain size of class  $n$ , with fraction  $f_n$ ,  $\theta_{cw}$  the instantaneous Shields parameter of dimensionless shear stress due to currents and waves and  $\mathbf{U}_{cw}$  the current due to current and waves, whose magnitude is given by (7.79).

The Shields parameter is computed here with the velocity due waves and currents and with a combined friction factor for waves and currents:

$$\theta_{cw}(t) = \frac{f_{cw} |\mathbf{U}_{cw}(t)|^2}{2(s-1)gd_{50}} \quad (7.93)$$

with

$$f_{cw} = \alpha f_c + (1 - \alpha) f_w \quad (7.94)$$

$$\alpha = \frac{u_{c\delta}}{u_{c\delta} + U_w} \quad (7.95)$$

$$f_c = 0.32 \left[ \ln \left( \frac{H}{z_{0s}} \right) - 1 \right]^{-2} \quad (7.96)$$

$$u_{c\delta} = \frac{u_*}{\kappa} \ln \left( \frac{30\delta}{z_0} \right) \quad (7.97)$$

where  $f_c$  is the friction factor due to currents and  $f_w$  the wave-related friction factor given by (7.18). The wave boundary layer thickness  $\delta$  is the same as the one for the Van Rijn (2007a) bed load formula.

The mean total load is obtained by decomposing the instantaneous sediment total load vector into components, along and transverse to the current direction, and integration over the wave period using (7.80) with  $q_b$  replaced by  $q_t$ . The components along the coordinate axes are obtained using (7.85) without slope effects (i.e.  $\alpha_s = 1$ ,  $\beta_s = 0$ ,  $\delta_s = 0$ ).

The integration over the wave period is executed in the code with the numerical integration method of Gaussian quadrature. For details about the applied method see Section 7.7.3. The number of data points needed in this method to reach a stable solution is optimised for the Madsen & Grant (1976) method. The fact that the transport is determined with the fifth power of the Shields number makes that the oscillating signal of the wave-current velocity results in a very peaked instantaneous transport. The consequence is that a relatively high number of points is needed to capture the sharp peak in the transport during maximal near-bed velocity. The optimal number for this method was determined at 15 points per wave period, see Figure 7.1.

The procedure is repeated for each class of sediment grain size defined by the user.

#### 7.5.4 Wu et al (2000)

The calculation procedure for bed load transport is described in Section 7.4.5. Adding the following equation for suspended load to the bed load formula results in the total sediment transport due currents only:

$$q_{s,n} = 0.262 \cdot 10^{-4} \sqrt{(s-1)gd^3} \left[ \left( \frac{\tau_b}{\tau_{cr,n}} - 1 \right) \frac{|\bar{\mathbf{u}}|}{w_{s,n}} \right]^{1.74} \quad (7.98)$$

Here, the settling velocity is calculated with equation (7.42) of Zhang & Xie (1993) and the critical shear stress using relation (7.33) from Wu *et al.* (2000).

#### 7.5.5 Van Rijn (2003)

In an unpublished paper Van Rijn (2003) describes approximate formulae for the bed and suspended load under combined currents and waves. A validity range is given for  $d_{50}$  between 0.1 and 0.5 mm sand, water depths between 0.25 and 20 m, current velocities between 0 and 2 m/s and relative wave heights  $H_s/H$  between 0 and 0.5. In this code, the bed load is not

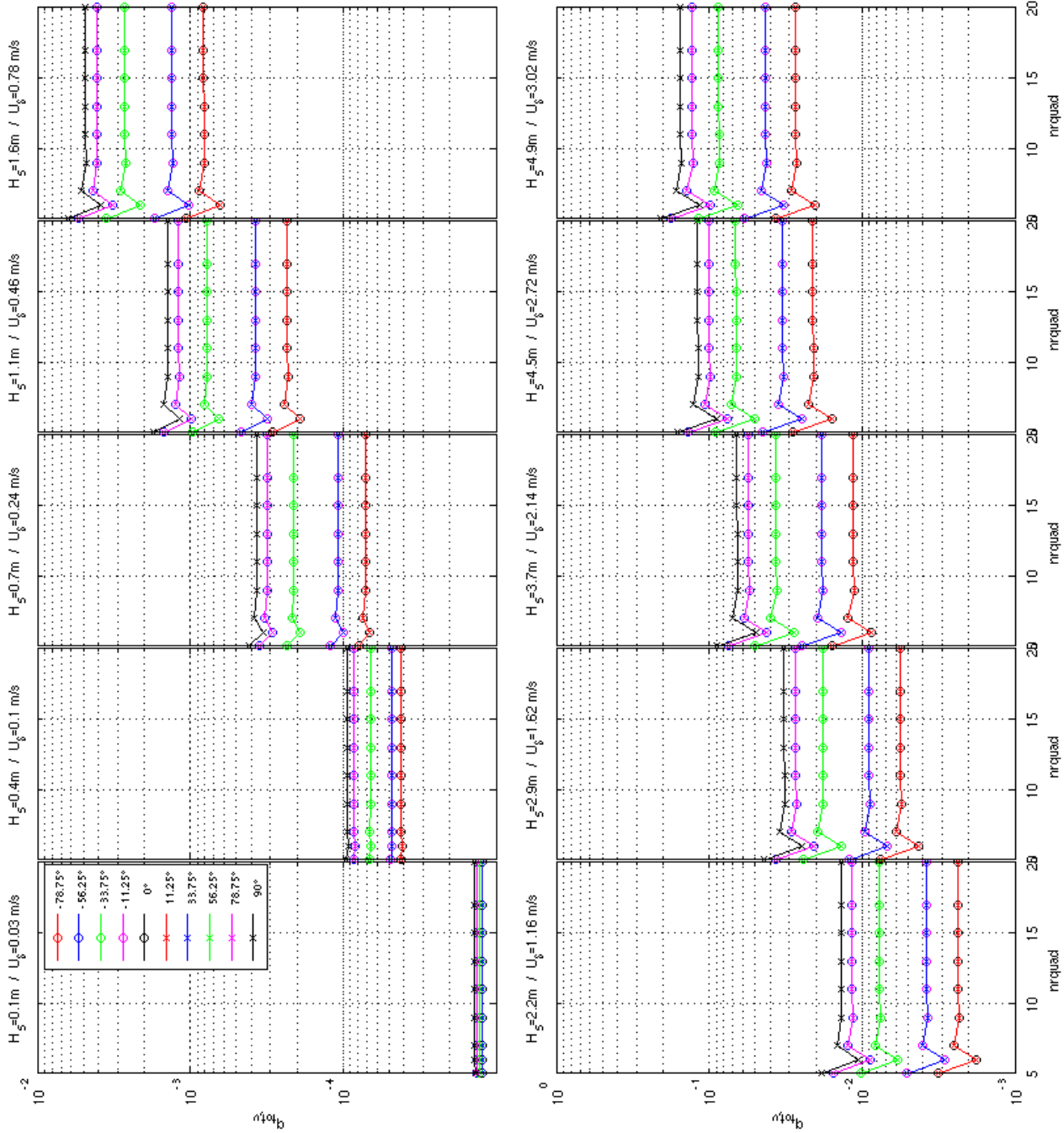


Figure 7.1: Convergence of Gauss-Legendre quadrature for wave period integration of instantaneous sediment transport. Different peak orbital velocities and wave-current angles are considered.

computed following the approximate formula, but with the fully integrated instantaneous oscillating transport, as described in Section 7.4.7. The suspended load, however, is computed using the explicit approximation formula of Van Rijn (2003).

The suspended sediment transport due to currents and waves is calculated by

$$q_s = q_{sc} + q_{sw} \quad (7.99)$$

which is the sum of the current-related suspended transport (with wave agitation), directed in the main current direction and the wave-related suspended transport in the wave propagation direction (associated with wave asymmetry). For simplicity, it is assumed in the following that there is only one grain size class in the sediment bed. In case of multiple fractions, the procedures are repeated for all classes, and the transports are multiplied with each class' fraction in bed.

#### 7.5.5.1 Current-related part

The current related part of the sediment transport is calculated as:

$$q_{sc} = (F_c + F_w)|\bar{\mathbf{u}}_c|Hc_a \quad (7.100)$$

Here,  $F_c$  and  $F_w$  are correction factors representing the depth-averaged concentration profile (relative to  $c_a$ ), for current and waves respectively. They are defined as follows

$$F_c = G(Z_c) \quad (7.101)$$

$$F_w = G(Z_w) \quad (7.102)$$

where the function  $G(Z)$  reads:

$$G(Z) = \frac{[(a/H)^Z - (a/H)^{1.2}]}{[(1.2 - Z)(1 - a/H)^Z]} \quad (7.103)$$

$$Z_c = \frac{w_s}{\beta_c K u_{*c}} + 2.5 \left( \frac{w_s}{u_{*c}} \right)^{0.8} \left( \frac{c_a}{c_{max}} \right)^{0.4} \quad (7.104)$$

$$Z_w = 4 \left( \frac{H}{H_{ref}} \right)^{0.6} \left( \frac{w_s T_P}{H_S} \right)^{0.8} \quad (7.105)$$

where  $\beta_c = 1 + 2(w_s/u_{*c})^2$ ,  $a$  the reference height (see Section 7.6.3.1),  $H_{ref}$  the reference water depth taken as 5 m,  $T_P$  the peak period of the waves,  $H_S$  the significant wave height and  $c_{max}$  the maximal concentration, which is taken equal to 0.65 for sand.

The appropriate expression for the reference concentration  $c_a$  is the one given in equation (7.125), but modified for wave-current interaction. The  $T$ -parameter (7.61) becomes then

$$T_n = \max \left( \frac{\tau_{b,cw}^e}{\tau_{cr,n}} - 1, 0 \right) \quad (7.106)$$

where  $\tau_{b,cw}^e = \tau_{b,c}^e + \tau_{b,w}^e$  is the time-(wave-) averaged effective bed shear stress and  $\tau_{cr,n}$  the critical bed shear stress for sediment class  $n$ .

The effective current-related bed-shear  $\tau_{b,c}^e$  is determined by:

$$\tau_{b,c}^e = \mu_c \alpha_{cw} \tau_{b,c} \quad (7.107)$$

here,  $\mu_c$  is the current-related efficiency factor, transforming the friction factor from total friction to grain-related friction, since the oscillating character of the flow does not permit it to feel bed forms with longer wavelength than the near-bed wave excursion. Also related to this effect is the wave-current interaction factor  $\alpha_{cw}$ . The former and latter parameters are defined as:

$$\mu_c = f_c^e / f_c = \left[ \frac{\log(H/30z_0)}{\log(H/1.5d_{50})} \right]^2 \quad (7.108)$$

$$\alpha_{cw} = \left[ \frac{\ln(90\delta_w/k_a)}{\ln(90\delta_w/k_s)} \right]^2 \left[ \frac{\ln(30H/k_s) - 1}{\ln(30H/k_a) - 1} \right]^2 \quad (7.109)$$

$$\alpha_{cw,max} = 1 \quad (7.110)$$

with  $\delta_w$  the wave boundary layer thickness and  $k_a$  the apparent roughness. For definitions, see Section 7.4.7. Note that the skin roughness height is set to  $1.5d_{50}$  which is the actual median grain size in the bed at each location from the particle size distribution in the highest bed layer.

The effective wave-related bed-shear  $\tau_{b,w}^e$  is in turn determined by:

$$\tau_{b,w}^e = \mu_w \tau_{b,w} \quad (7.111)$$

In the 2003 version of the van Rijn model,  $\mu_w$  is given by  $0.125(1.5 - H_s/H)^2$ . This parameter will be different in the 2007 version described in the next section. The bed shear stresses due to waves and due to currents are determined using the same friction factors as in the bed load transport model, see Section 7.4.7.

### 7.5.5.2 Wave-related part

This part of the suspended load – with equal values in the 2003 and 2007 versions of the van Rijn model – is dependent on the wave asymmetry in

terms of onshore and offshore directed peak velocities. These velocities have been computed using the method of Grasmeyer & Van Rijn (1998). The wave propagation directed suspended transport is given by:

$$q_{sw,n} = \gamma f_n U_A L_T \quad (7.112)$$

with

$q_{sw,n}$	volumetric suspended sediment transport of class $n$ [ $\text{m}^2/\text{s}$ ]
$U_A = (U_{\delta,f}^4 - U_{\delta,b}^4) / (U_{on}^3 + U_{off}^3)$	velocity asymmetry [m/s]
$L_T = 0.007 d_n M_e$	suspended sediment load [m]
$M_e = (v_e - v_c)^2 / ((s-1)gd_n)$	Sediment mobility number due to waves and currents
$U_{\delta,f}$	Near-bed orbital velocity peak in onshore (forward) direction [m/s]
$U_{\delta,b}$	Near-bed orbital velocity peak in offshore (backward) direction [m/s]
$v_e = \sqrt{U^2 + U_{\delta,f}^2}$	effective velocity due to currents and waves [m/s]
$v_c = 0.19 d_n^{0.1} \ln(2h/d_n)$	critical velocity for $0.1 \text{ mm} < d_n < 0.5 \text{ mm}$ [m/s]
$v_c = 8.50 d_n^{0.6} \ln(2h/d_n)$	critical velocity for $d_n > 0.5 \text{ mm}$ [m/s]
$d_n$	grain size of sediment class $n$ [m]
$\gamma$	phase lag coefficient = 0.2

A number of assumptions have been made during the development of this simplified approximative formulation: a bed roughness length  $k_s$  of 0.03m, wave-current angle of 90 degrees, temperature of 15°C; salinity of 30 ppt. The onshore and offshore peaks of the asymmetric wave boundary velocities  $U_{\delta,f}$  and  $U_{\delta,b}$  are computed using the method of Grasmeyer & Van Rijn (1998), which will not be reproduced here.

Effectively, in this method, the critical condition for initiation of motion is defined specifically. The methods used in other parts of the code for determination of critical shear are not applied for the wave part of suspended load.

### 7.5.6 Van Rijn (2007a)

The Van Rijn (2007a) method is very similar to the 2003 version, with a different formulation for some of the parameters: the turbulence damping factor and the wave-related efficiency factor.

Due to the relation adopted for turbulence damping due to high sediment concentrations, the modified Rouse number in the approximative formulation for current-related suspended load ( $Z_c$  in equation (7.104)) takes a different form

$$Z_c = \frac{w_s}{\beta_c \kappa u_* \phi_d} \quad (7.113)$$

Here,  $\phi_d$  is the turbulence damping function used, originally varying with concentration, but here evaluated for the reference concentration

$$\phi_d = 1 + (c_a/c_{max})^{0.8} - 2(c_a/c_{max})^{0.4} \quad (7.114)$$

The wave-related efficiency factor  $\mu_w$  takes a different form in this formulation. It is made dependent of the dimensionless particle diameter rather than the wave height relative to the water depth:

$$\mu_w = \begin{cases} 0.35 & \text{if } d_* < 2 \\ 0.7/d_* & \text{if } 2 < d_* < 5 \\ 0.14 & \text{if } 5 < d_* \end{cases} \quad (7.115)$$

### Implementation

The type of total load formulation is selected with the following switches

`iopt_sed_toteq` Method for total load transport

- 1: Engelund & Hansen (1967). The precise form is also determined by the switch `iopt_sed_eha`.
- 2: Ackers & White (1973)
- 3: Madsen & Grant (1976). This equation includes wave effects.
- 4: Wu *et al.* (2000). Total load is calculated as the sum of suspended and bed load.
- 5: Van Rijn (2003). This equation includes wave effects and total load is the sum of suspended and bed load.
- 6: Van Rijn (2007a). This equation includes wave effects and total load is the sum of suspended and bed load.

`iopt_sed_eha` Switch to select the type of formulation in the Engelund-Hansen total load equation (7.86)

- 1: original form
- 2: Chollet & Cunge (1979) form as function of  $\theta_*$

## 7.6 Suspended sediment transport

### 7.6.1 Three-dimensional sediment transport

In COHERENS multiple sediment fractions can be taken, each with its own diameter, settling velocity, critical shear stress, . . . . In an orthogonal curvilinear coordinate system, the sediment concentration of each sediment fraction is governed by the following advection-diffusion equation for the volumetric sediment concentration fraction  $c$  (with the subscript  $n$  omitted):

$$\begin{aligned} \frac{1}{h_3} \frac{\partial}{\partial t} (h_3 c) + \frac{1}{h_1 h_2 h_3} \left( \frac{\partial}{\partial \xi_1} (h_2 h_3 (u - \tilde{u}_s) c) + \frac{\partial}{\partial \xi_2} (h_1 h_3 (v - \tilde{v}_s) c) \right) + \\ \frac{1}{h_3} \frac{\partial}{\partial s} (h_3 (\omega - w_s) c) = \\ \frac{1}{h_3} \frac{\partial}{\partial s} \left( \frac{D_V}{h_3} \frac{\partial c}{\partial s} \right) + \frac{1}{h_1 h_2 h_3} \left( \frac{\partial}{\partial \xi_1} \frac{D_H h_2 h_3}{h_1} \frac{\partial c}{\partial \xi_1} + \frac{\partial}{\partial \xi_2} \frac{D_H h_1 h_3}{h_1} \frac{\partial c}{\partial \xi_2} \right) + \frac{Q_{sed}}{h_1 h_2 h_3} \end{aligned} \quad (7.116)$$

where  $D_V = \lambda_T^s$ ,  $D_H = \lambda_H$  are the vertical, respectively horizontal diffusion coefficients for sediments,  $w_s$  the settling velocity, defined in Section 7.3.3 and  $Q_{sed}$  a user defined source term. More information on the used coordinate systems can be found in Section 4.1. In the 3-D version of the transport equation, erosion and depositions of sediment enter through the boundary conditions (Section 7.6.3).

Equation (7.116) is the same as the scalar transport equation (4.76) for a scalar quantity  $\psi$  except that the equation now contains an extra vertical advective term for the settling velocity. Since the settling velocity is directed along the vertical which does not coincide with the transformed vertical direction normal to the iso- $\sigma$  surfaces, the horizontal advective currents need to be adjusted by adding a correction for settling

$$\tilde{u}_s = - \frac{w_s}{h_1} \frac{\partial z}{\partial \tilde{\xi}_1} \Big|_s = - w_s \frac{h_3}{h_1} \frac{\partial s}{\partial \xi_1} \Big|_z \quad (7.117)$$

$$\tilde{v}_s = - \frac{w_s}{h_2} \frac{\partial z}{\partial \tilde{\xi}_2} \Big|_s = - w_s \frac{h_3}{h_2} \frac{\partial s}{\partial \xi_2} \Big|_z \quad (7.118)$$

### 7.6.2 Two-dimensional sediment transport

As explained in Section 4.3.2, the advection-diffusion equations for sediments can also be solved in depth-averaged form. In that case, erosion  $E$  and deposition  $D$ , used to determine the bottom boundary condition in the 3-D



case (see below), are modeled by source terms in the depth averaged sediment concentration equation

$$\frac{\partial \bar{c}}{\partial t} + \frac{1}{h_1 h_2} \left( \frac{\partial}{\partial \xi_1} h_2 \bar{c} \bar{u} + \frac{\partial}{\partial \xi_2} h_1 \bar{c} \bar{v} \right) = \frac{1}{h_1 h_2} \left( \frac{\partial}{\partial \xi_1} \bar{D}_H \frac{h_2}{h_1} \frac{\partial \bar{c}}{\partial \xi_1} + \frac{\partial}{\partial \xi_2} \bar{D}_H \frac{h_1}{h_2} \frac{\partial \bar{c}}{\partial \xi_2} \right) + \frac{E - D}{H} \quad (7.119)$$

For details see Section 4.3.2.

### 7.6.3 Erosion and deposition

Erosion and deposition have a double influence. On the one hand, the erosion and deposition provide the bottom boundary conditions for the advection-diffusion equation of sediment transport. On the other hand, they provide an important source term in the equation for the bed-level elevation for the morphology. In this section, the modeling of the erosion and deposition terms is described. This is first done for three-dimensional situations, where the erosion and deposition terms are introduced through the near-bed boundary conditions. These terms are described separately for sand and mud, because they are calculated in a different way. This is followed by a description of erosion and deposition in two-dimensional situations, where they are represented as a source term.

#### 7.6.3.1 Erosion and deposition of sand in 3-D

The following equation is used as the boundary condition near the bed for the suspended sediment transport equation (taken at a reference height  $a$  above the bed)

$$-D_V(a) \frac{\partial c_n}{\partial z}(a) - w_{s,in}(a) c_n(a) = E_n - D_n \quad (7.120)$$

Here,  $E_n$  is the erosion of sediment fraction  $n$ ,  $D_n$  the deposition,  $c_n$  the volumetric sediment concentration, and  $w_{s,n}$  the settling velocity of fraction  $n$ . Note that because the volume concentrations are used in COHERENS, the dimensions of the erosion and deposition are  $m/s$ .

The deposition flux is given as

$$D_n = w_{s,n}(a) c_n(a) \quad (7.121)$$

Using this expression, the bottom boundary conditions reduces to the following Neumann boundary condition:

$$-D_V(a) \frac{\partial c_n}{\partial z} \Big|_a = E_n \quad (7.122)$$

Garcia & Parker (1991) showed that the flux can be expressed as a non-dimensional flux  $E_* \equiv E/w_s$  and that in situations not too far from equilibrium, this non-dimensional entrainment is equal to the near bed reference concentration (i.e.  $E_* = c_a$ ). Therefore one can write:

$$-D_V(a) \frac{\partial c_n}{\partial z} \Big|_a = f_n w_{s,n}(a) c_{a,n} \quad (7.123)$$

Here  $c_{a,n}$  is the near-bed reference (equilibrium) sediment concentration at a reference height  $a_n$  for sediment size class  $n$  with fractional amount  $f_n$  at the bed.

In COHERENS, the near bed reference concentration for sand is calculated either by the method of Smith & McLean (1977) or by the one in Van Rijn (1984a), which can be selected with the switch `iopt_sed_bbc`.

The near-bed boundary condition of Smith & McLean (1977) is given for each fraction  $n$  by:

$$\begin{aligned} c_{a,n} &= 0.0024 c_{max} \frac{T_n}{1 + 0.0024 T_n} \\ a &= k_s + 26.3(\theta_n - \theta_{cr,n}) d_n \\ k_s &= 30 z_0 \end{aligned} \quad (7.124)$$

Here,  $c_{max}$  is the maximum possible concentration, i.e. the concentration of a bed packed with sediment,  $T_n$  the normalised excess shear stress defined by (7.61),  $\theta$  the Shield parameter (7.27),  $k_s$  the skin roughness height and  $z_0$  the skin roughness length.

The near bed boundary condition of Van Rijn (1984a) is given by:

$$c_{a,n} = 0.015 \frac{d_n T_n^{1.5}}{a_n d_{*n}^{0.3}} \quad (7.125)$$

with  $d_*$  defined by (7.29). The reference level  $a$  (either half the size of the dunes or the roughness length scale  $k_s$ ) is limited to be between  $0.01H$  and  $0.1H$ .

It may occur that the reference level  $a_n$  is not located within the bottom grid cell where the bottom boundary condition is applied. The methods used in COHERENS to overcome this problem are described in Section 7.7.1. The influence of waves on the erosion of sediment is included in this equation only by the change in the bed shear stress due to the near bed wave motion using the equation of Soulsby (1997).

### 7.6.3.2 Erosion of cohesive sediment in 3-D

Erosion of cohesive sediments is calculated with the equation based on the data of Partheniades (1965)

$$E_n = f_n M \left( \frac{\tau_b - \tau_{cr,n}}{\tau_{cr,n}} \right)^{n_p} \quad \text{for } \tau_b > \tau_{cr,n} \quad (7.126)$$

Here  $E_n$  is the erosion rate,  $M$  is an erosion rate parameter (m/s),  $\tau_b$  the bed shear stress,  $\tau_{cr}$  the critical shear stress threshold for erosion and  $n_p$  an empirical parameter (normally equal to unity). The parameters  $M$  and  $\tau_{cr,n}$  need to be determined, which can be difficult in practice, because they depend on the material, its consolidation as well as biological and chemical characteristics of the bed, thus they can vary in space and time and  $f_n$  is the amount of this fraction in the bed. In COHERENS, the critical shear stress is determined using one of the methods, described in Section 7.3.2. Note that the unit of  $M$  (m/s) and  $\tau_{cr,n}$  ( $\text{m}^2/\text{s}^2$ ) in the code is different from usual, where the units of these quantities are respectively  $\text{kg}/\text{m}^2/\text{s}$  and  $\text{N}/\text{m}^2$ .

The implementation of the three-dimensional erosion of cohesive sediment differs from sand in that no reference concentration is calculated, and no reference height has to be specified. Just as for sand, wave effects are only included through the changes in the bed shear stress.

### 7.6.3.3 Erosion-deposition of sand in 2-D

In two dimensional calculations, the erosion and deposition of sand are calculated from the difference between the depth averaged concentrations and the depth averaged equilibrium concentration. A time scale  $T_{e,n}$  is used in which the sediment concentration adapts to the equilibrium concentration. This represents the fact that deposition depends on the near-bed sediment concentration, rather than the depth averaged value and thus that the sediment concentration profile needs some time to adapt to the equilibrium profile (e.g. Galappatti & Vreugdenhil, 1985). The equations that are used for the erosion deposition term are

$$E_n - D_n = \frac{H}{T_{e,n}} (\bar{c}_{e,n} - \bar{c}_n) \quad (7.127)$$

The equilibrium concentration  $\bar{c}_e$  can be calculated in different ways, which are selected by the switch `iopt_sed_ceqeq`. The default option is to calculate the equilibrium concentration by taking the depth average of the Rouse profile numerically (see Section 7.7.3 for the numerical method), which

is given by

$$\frac{c_{eq}}{c_a} = \left[ \frac{H - \tilde{z}}{\tilde{z}} \frac{a}{H - a} \right]^R \quad (7.128)$$

where  $a$  is the reference level,  $H$  the total water depth,  $\tilde{z} = z + h$  the height above the sea bed,  $c_a$  the reference concentration,  $R = w_s / \beta \kappa u_*$  the Rouse number,  $w_s$  the particle fall velocity,  $u_*$  the skin shear velocity,  $\beta$  the inverse of the Prandtl-Schmidt number (see Section 7.6.4) and  $\kappa$  the von Karman constant. The exponent expresses the relative importance of the suspended load, where it is equal to about 5 for bed load only and decreases to unity for suspended load up to the surface (Van Rijn, 1993). The reference concentration is defined in Section 7.6.3.1.

Another way to calculate the depth averaged sediment concentrations is by using one of the formulae for suspended transport, described in Section 7.5. The depth averaged equilibrium concentration is then given by

$$\bar{c}_e = \frac{q_{s,e}}{|\mathbf{U}|} \quad (7.129)$$

Here  $q_{s,e}$  is the equilibrium suspended sediment transport and  $|\mathbf{U}|$  the magnitude of the depth-integrated current (in  $\text{m}^2/\text{s}$ ).

The adaptation time scale  $T_e$  is given by:

$$T_e = \frac{H}{w_s} T_* \quad (7.130)$$

Here,  $T_*$  is a non-dimensional correction factor, which depends on the shape of the sediment concentration profile, parametrized by the Rouse number  $R$  defined below. For COHERENS, this relation was obtained by performing simulations with COHERENS in water column (1DV) mode and determining the adaptation time scale from the data (see Figure 7.2). Through this data an eight order polynomial was fitted:

$$T_* = p_1 R^8 + p_2 R^7 + p_3 R^6 + p_4 R^5 + p_5 R^4 + p_6 R^3 + p_7 R^2 + p_8 R + p_9 \quad (7.131)$$

with:

$$R = \min(w_{sb}/u_*, 1.14) \quad (7.132)$$

and

$$\begin{array}{lll} p_1 = 67.342 & p_2 = -321.0 & p_3 = 614.14 \\ p_4 = -592.75 & p_5 = 292.15 & p_6 = -62.141 \\ p_7 = 3.667 & p_8 = -2.2571 & p_9 = 0.97978 \end{array} \quad (7.133)$$

and  $w_{sb}$  the settling velocity at the bottom.

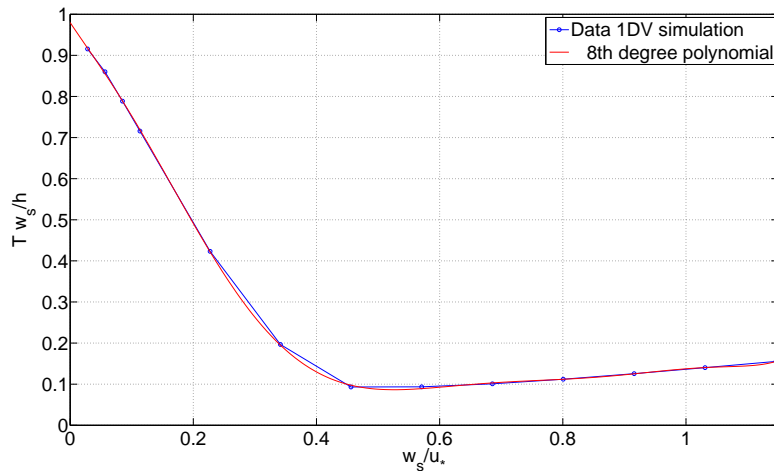


Figure 7.2: Numerically determined time scale and fitted polynomial

#### 7.6.3.4 Erosion-deposition of cohesive sediment in 2-D

For cohesive sediments, the erosion and deposition rates in 2-D are obtained in similar way as for sand, using an equilibrium time scale and a depth averaged concentration (Section 7.6.3.3). For cohesive sediment, the only way in which the depth averaged equilibrium concentration can be calculated is by averaging the Rouse profile (equation 7.128). However, a near-bed reference concentration and reference location are needed to obtain the depth averaged concentrations. The near bed reference concentration is obtained from the near bed erosion rate  $E$  using

$$c_a = \frac{E}{w_s(a)} \quad (7.134)$$

The near bed reference location  $a$  is, in the 2-D case, an extra tunable parameter `height_c_cst`. This is not very different from the usual approach to use a threshold of deposition as an extra tunable parameter for simulating cohesive sediment transport. However, the present approach has the advantage that unphysical results that may result from the threshold of deposition method (such as the unlimited erosion leading to unphysically high sediment concentrations) are avoided. The same time scale is used as for the calculation of depth averaged sand transport.

## 7.6.4 Sediment diffusivity

### 7.6.4.1 Without wave effects

It has been found experimentally that the diffusivity of sediment is not always equal to the eddy diffusivity. For practical purposes, the sediment diffusivity  $D_V$  is assumed to be proportional to the eddy viscosity

$$D_V = \beta \nu_T \quad (7.135)$$

Here,  $\beta$  is the inverse of the Prandtl-Schmidt number  $\sigma_T$  and  $\nu_T$  the eddy viscosity defined in Section 4.4. Different relations have been proposed for the  $\beta$  factor. In COHERENS,  $\beta$  can be either a constant or calculated with the equation of Van Rijn (1984b). Based on the work of (Coleman, 1970), he found that it can be taken as a function of the settling velocity

$$\beta = 1 + 2 \left( \frac{w_s}{u_*} \right)^2 \quad (7.136)$$

For practical applications,  $\beta$  is limited to values between 1 and 1.5.

### 7.6.4.2 With waves effects

The vertical sediment diffusivity due to combined waves and currents  $D_{V,cw}$  is calculated from the sediment diffusivity due to current and waves using (Van Rijn, 2007b)

$$D_{V,cw} = \sqrt{(D_{V,c})^2 + (D_{V,w})^2} \quad (7.137)$$

Here,  $D_{V,c} = \beta \nu_T$  is the sediment diffusivity in the absence of waves and  $D_{V,w}$  an additional component due to waves.

The wave related sediment diffusivity  $D_{V,w}$  is calculated by separate expressions for the upper half of the water column and for the near-bed mixing layer. Between the two layers, a linear interpolation is performed (Van Rijn, 2007b)

$$\begin{aligned} D_{V,w} &= D_{V,w,max} = 0.035 \gamma_{br} H H_s / T_p \leq 0.05 \text{ m}^2/\text{s} & \text{for } 0.5H \leq \tilde{z} \\ D_{V,w} &= D_{V,w,bed} + (D_{V,w,max} - D_{V,w,bed}) \left[ \frac{\tilde{z} - \delta_s}{0.5H - \delta_s} \right] & \text{for } \delta_s < \tilde{z} < 0.5H \\ D_{V,w} &= D_{V,w,bed} = 0.018 \gamma_{br} \beta_w \delta_s U_{\delta,r} & \text{for } \tilde{z} \leq \delta_s \end{aligned} \quad (7.138)$$

where  $\tilde{z}$  is the distance above the sea bed,  $U_{\delta,r}$  the near-bed peak orbital velocity  $U_w$ , given by (7.8),  $T_p$  the peak wave frequency and  $\delta_s$  the thickness of the mixing layer

$$\delta_s = 0.072 \gamma_{br} A_\delta (A_\delta / k_w)^{-0.25} \quad (7.139)$$

Here,  $A_\delta$  is the near-bottom orbital excursion  $A_b$ , given by (7.13),  $k_w$  the wave-related bed roughness height,

$$\gamma_{br} = 1 + (H_s/H - 0.4)^{0.4} \quad (7.140)$$

a coefficient related to wave breaking ( $= 1$  for  $H_s/H \leq 0.4$ ), and

$$\beta_w = 1 + 2(w_s/u_{*,w})^2 \quad (7.141)$$

where  $u_{*w}$  is the wave-related bed-shear velocity.

### 7.6.5 Boundary conditions

Bottom boundary conditions for sediment are defined through erosion and deposition and discussed in Section 7.6.3.

At the free surface, a vertical zero-flux condition is used for the suspended sediment:

$$D_V \frac{\partial c_n}{\partial z} + w_{s,n} c_n = 0 \quad (7.142)$$

The lateral boundary conditions are the same as the ones in discussed in Section 4.10.2.2.

### Implementation

The following switches are used for the transport of suspended sediment:

`iopt_sed_mode` Type of mode for applying the sediment transport model

- 1: bedload transport only computed by a formula, which is determined by `iopt_sed_bedeq`
- 2: suspended load transport only (computed with the advection-diffusion equation)
- 3: bedload and suspended transport (i.e. option 1 and 2 together)
- 4: total load transport computed with a formula, which is determined by `iopt_sed_toteq`

`iopt_sed_nodim` The number of dimensions used in the sediment formulation.

- 2: depth averaged transport<sup>3</sup>
- 3: 3-D sediment transport.

---

<sup>3</sup>Note that `iopt_sed_nodim` is always set to 2 if `iopt_grid_nodim = 2`.

`iopt_sed_type` The type of sediment that is used

- 1: sand (non-cohesive)
- 2: mud (cohesive)

`iopt_sed_bbc` The type of equation for bed boundary condition at the sea bed

- 0: no bed boundary conditions (no flux to and from the bed)
- 1: Smith & McLean (1977)
- 2: Van Rijn (1984a)
- 3: Partheniades (1965)

`iopt_sed_ceqeq` The type of model for determining the equilibrium sediment concentration

- 1: numerical integration of the Rouse profile
- 2: Using  $q_t/U$  determined with the equation of Engelund & Hansen (1967). The precise form is also determined by the switch `iopt_sed_eha`.
- 3: using  $q_t/U$  determined with the equation of Ackers & White (1973).
- 4: Using  $q_s/U$  determined with the equation of Van Rijn (2003). This formulation is very similar to Van Rijn (1984b), but takes wave stresses into account.
- 5: using  $q_s/U$  determined with the equation of Wu *et al.* (2000)

`iopt_sed_beta` The type of equation used for  $\beta$ , the ratio between the eddy viscosity and eddy diffusivity

- 1:  $\beta = 1$ .
- 2: user defined value of  $\beta$  (`beta_cst`).
- 3: Van Rijn (1984b).

`iopt_sed_wave_diff` Selects the turbulent diffusion coefficient due to waves.

- 0: No diffusion coefficient
- 1: According to Van Rijn (2007b)

## 7.7 Numerical methods

This section describes the numerical methods that are specifically used in the sediment transport module. The methods are generally the same as the



ones for a scalar quantity, as described in Section 5.5. However, there are some differences, mainly related to the modeling of erosion and deposition, which are discussed here.

## 7.7.1 Erosion-deposition

### 7.7.1.1 Three-dimensional sediment transport

In three-dimensional simulations, erosion and deposition come into play through the near-bed boundary condition. For mud, the erosion rate is given directly by the equation of Partheniades (1965). For sand, the erosion  $E$  is calculated from the near bed reference concentration  $c_a$  at a height  $a$  above the bed using  $E = w_s c_a$ . However, the height  $a$  does not normally coincide with the used computational grid. Hence, a transformation has to be made, such that the erosion is calculated at a location coinciding with the computational grid.

In COHERENS, three methods are available to do this, which can be set with the switch `iopt_sed_bbc_type`. The first two options are based on the method used in the EFDC model. In this method, the flux is calculated based on an averaged of either the lowest cell (`iopt_sed_bbc_type = 1`) or the cell in which  $a$  lies (`iopt_sed_bbc_type = 2`). At a given reference level  $a$ , the net upward flux is given by

$$F(a) = w_s(c_a - c) \quad (7.143)$$

Integrating this expression over the bottom cell gives the bottom flux into that bottom cell,  $F_0$ :

$$F_0 = w_s(\bar{c}_{eq} - c_1) \quad (7.144)$$

where  $\bar{c}_{eq}$  is the averaged equilibrium concentration for the bottom cell,  $c_1$  is the sediment concentration in the bottom cell. The determination of  $\bar{c}_{eq}$  can be done by solving equation (7.146) below for  $F_0$ , and averaging over the bottom cell by integration between  $z_0$  and  $z_1$ , i.e. the W-nodes below and above the bottom cell. In case the reference level  $a$  is located above the near-bed cell and `iopt_sed_bbc_type = 2`, the  $n^{\text{th}}$  cell is integrated (the ‘reference layer’), from  $z_{n-1}$  up to  $z_n$ , and the  $c_1$  term in equation (7.144) becomes  $c_n$ , yielding:

$$F_0 = w_s \left( \frac{c_a}{\Delta z_n} \int_{z_{n-1}}^{z_n} \left( \frac{a}{z} \right)^R dz - \frac{1}{\Delta z_n} \int_{z_{n-1}}^{z_n} c dz \right) \quad (7.145)$$

where  $\Delta z_n$  is the thickness of the  $n^{\text{th}}$  cell, with the reference level  $a$  located in the  $n^{\text{th}}$  cell.

Herein, the following expression is used for the non-equilibrium near bed sediment profile (derived assuming a linear varying eddy viscosity near the bed)

$$c = c_a \left( \frac{a}{z} \right)^R - \frac{F_0}{w_s} \quad (7.146)$$

Combining this expression with equation (7.144) gives

$$\bar{c}_{eq} = \frac{c_a}{\Delta z_n} \int_{z_{n-1}}^{z_n} \left( \frac{a}{z} \right)^R dz \quad (7.147)$$

For  $R \neq 1$ , this gives

$$\bar{c}_{eq} = \frac{a^R c_a}{\Delta z_n (1 - R)} [z_n^{1-R} - z_{n-1}^{1-R}] \quad (7.148)$$

and for  $R = 1$

$$\bar{c}_{eq} = \frac{a c_a}{\Delta z_n} [\ln z_n - \ln z_{n-1}] \quad (7.149)$$

The range of values for the Rouse number  $R$  are limited by the theoretical maximum of 2.5, since no suspension can exist when the settling velocity is larger than the shear velocity. When the method of cell-averaged reference concentration described above is used while the reference level is located within the bottom cell, the integrated profile (equations (7.148)-(7.149)) starts from the bottom of the near-bed cell, at level  $z_0$ . Since the Rouse profile is not valid that close to the bed, another lower limit for integration is used, with the minimum set at the Nikuradze roughness  $k_s = 30z_0$ . The level  $z_0$ , in turn, is the level at which the assumed logarithmic velocity profile becomes equal to zero.

This method with `iopt_sed_bbc.type = 2` can give very accurate results, provided that a high resolution is used near the bed. However, when the resolution is relatively low (and thus the cells near the bed are relatively large), the averaging of the profile leads to an overestimation of the concentrations in the lowest cells and therefore in sediment concentrations that are too high. Therefore, this method is not recommended for practical applications. Instead, another method should be used by setting `iopt_sed_bbc.type = 3`. In this method, the Rouse profile (equation (7.128)) is used to calculate the sediment concentration from the given boundary condition at the first C-node above the bed.

The deposition flux is just an advection term from the lowest cell. The type discretisation of this term at the bed is determined by the switch `iopt_scal_depos`. The options are

- No deposition flux (all sediment remains inside the computation domain). This is useful for simulating some laboratory experiments.
- First order discretisation
- Second order discretisation

The first order discretisation of the deposition flux  $D$  is an upwind discretisation that is given by:

$$D = w_1^w c_1^c \quad (7.150)$$

The second order discretisation of the deposition flux uses linear extrapolation to obtain an estimation of the sediment concentration at the lowest W-node. It is given by

$$D_{ij} = w_{s;ij1} \left[ \left( 1 + \frac{h_{3;ij1}^c}{2h_{3;ij2}^w} \right) c_{ij1} - \frac{h_{3;ij1}^c}{2h_{3;ij2}^w} c_{ij2} \right] \quad (7.151)$$

### 7.7.1.2 Time integration of sediment transport

In 2-D mode, the source/sink term is integrated semi-implicitly in time

$$E - D = \frac{H^{n+1}}{T_e} (\bar{c}_e - \theta_v \bar{c}^{n+1} - (1 - \theta_v) \bar{c}^n) \quad (7.152)$$

In case of 3-D simulations, erosion is taken explicitly whereas deposition is integrated semi-implicitly with the implicitity factor  $\theta_a$  which is the same as the one used for the vertical advective term in scalar transport equations (see Section 5.5.3).

## 7.7.2 Bed slope factors

Using the notations of Section 7.4.9, one has

$$\begin{aligned} \cos(\phi_c + \delta_s) &= \cos \phi_c \cos \delta_s - \sin \phi_c \sin \delta_s \\ \sin(\phi_c + \delta_s) &= \sin \phi_c \cos \delta_s + \cos \phi_c \sin \delta_s \end{aligned} \quad (7.153)$$

Letting

$$\beta_* = \beta_s \frac{\partial h}{\partial n} = \tan \delta_s \quad (7.154)$$

one has

$$\cos \delta_s = \cos \arctan \beta_* = \frac{1}{\sqrt{1 + \beta_*^2}}$$

$$\sin \delta_s = \sin \arctan \beta_* = \frac{\beta_*}{\sqrt{1 + \beta_*^2}} \quad (7.155)$$

so that

$$\begin{aligned} \cos(\phi_c + \delta_s) &= \frac{\cos \phi_c - \beta_* \sin \phi_c}{\sqrt{1 + \beta_*^2}} \\ \sin(\phi_c + \delta_s) &= \frac{\sin \phi_c + \beta_* \cos \phi_c}{\sqrt{1 + \beta_*^2}} \end{aligned} \quad (7.156)$$

### 7.7.3 Gaussian-Legendre quadrature

In COHERENS, numerical integration needs to be performed in two cases:

- to calculate phase averages from instantaneous values in the load formulae of Madsen & Grant (1976) and Van Rijn (2003, 2007b)
- to determine the equilibrium concentration by taking the vertical average of the Rouse profile (7.128).

The integrals are calculated by applying Gaussian quadrature. The integral of a function  $f(x)$  between  $a$  and  $b$  is calculated as:

$$\int_a^b f(x) dx \simeq \frac{b-a}{2} \sum_{i=1}^n w_i f\left(\frac{b-a}{2}x_i + \frac{a+b}{2}\right) \quad (7.157)$$

Here,  $n$  is the number of points in the integration  $x_i$  are the locations of the nodes where the function is evaluated and  $w_i$  are the weighting factors. The locations  $x_i$  are the roots of the  $n$ -th order Legendre polynomial  $P_n$  and  $w_i$  is determined by

$$w_i = \frac{2}{(1-x_i)^2} \left( \frac{dP_n(x_i)}{dx} \right)^{-2} \quad (7.158)$$

The locations and weight factors are obtained by calling the routine `gauss_quad` with, usually, a user-applied value  $n$ .

The phase average of a function  $f(\varphi)$  with  $\varphi = \omega t$ , is then given by

$$F = \frac{1}{2\pi} \int_0^{2\pi} f(\varphi) d\varphi \simeq \frac{1}{2} \sum_{i=1}^n w_i f(\varphi_i) \quad (7.159)$$

with  $\varphi_i = \pi(1 + x_i)$ .

The equilibrium concentration  $\bar{c}_e$  is given by

$$\bar{c}_e \simeq \frac{c_a}{H} \int_a^H \left[ \frac{H-z}{z} \frac{a}{H-a} \right]^R dz \quad (7.160)$$

Letting  $\phi = \ln(z/H)$ ,  $a_* = a/H$  and applying (7.157) one obtains

$$\begin{aligned} \bar{c}_e &= c_a \left( \frac{a_*}{1-a_*} \right)^R \int_{\ln a_*}^0 e^\phi (e^{-\phi} - 1)^R d\phi \\ &\simeq -\frac{c_a}{2} \left( \frac{a_*}{1-a_*} \right)^R \ln a_* \sum_{i=1}^n w_i e^{\phi_i} (e^{-\phi_i} - 1)^R \end{aligned} \quad (7.161)$$

with  $\phi_i = \frac{1}{2}(1-x_i) \ln a_*$ .

#### 7.7.4 Bartnicki filter

Negative concentrations may arise by application of the bottom boundary conditions. In order to prevent negative sediment concentrations, a Bartnicki (1989) filter can be used with the switch `iopt_sed_filter`. This filter eliminates negative sediment concentrations, while conserving the total sediment volume. It is based on the assumption that the amount of negative concentration is much smaller than the amount of positive concentrations. The Bartnicki filter performs, at each time step, the following steps iteratively until no negative concentrations exist in the model.

- The total volume of negative concentrations is determined:  $M_- = \sum_{ijk} c_{ijk} (c_{ijk} < 0) \Delta V_{ijk}$  where  $\Delta V_{ijk} = h_{1;ij} h_{2;ij} h_{3;ijk}$  is the volume of a grid cell.
- The total volume of the cells with a positive concentration (larger than 0) is determined:  $V_+ = \sum_{ijk} (c_{ijk} > 0) \Delta V_{ijk}$ .
- The average difference volume is determined:  $M_+ = M_- / V_+$ .
- The concentration in cells with a negative concentration is set to zero.
- $M_+$  is added to the concentration in cells with a positive concentration.

#### Implementation

Numerical methods, described in this section, are selected with the following switches:

`iopt_sed_bbc_type` Selects the method to transpose the near bed boundary condition to the computational grid. It is strongly recommended not to change the default value of 3.

- 1: EFDC method applied to lowest cell (not recommended)
- 2: EFDC method applied to the first the cell above (not recommended)
- 3: Using the Rouse profile

`iopt_sed_filter` Disables (0) or enables (1) the application of the Bartnicki filter to prevent the occurrence of negative concentrations .