Appendix C

Discretisation of the Coriolis force

The Coriolis terms in the 2-D and 3-D momentum equations are discretised in time using a "quasi"-implicit formulation. The time discretised equations are written as

$$\frac{u^{n+1}-u^n}{\Delta t} - f\Big(\theta_c v^{n+1} + (1-\theta_c)v^n\Big) = F_1 \tag{C.1}$$

$$\frac{v^{n+1}-v^n}{\Delta t} + f\Big(\theta_c u^{n+1} + (1-\theta_c)u^n\Big) = F_2 \tag{C.2}$$

where F_i represent all other (explicit and implicit) terms (pressure gradient, advection, diffusion, tidal force). The solution proceeds in two steps

1. Equations (C.1–(C.2) are solved explicitly for the Coriolis force ($\theta_c = 0$). This gives

$$u^* = u^n + \Delta t(F_1 + fv^n) \tag{C.3}$$
$$v^* = v^n + \Delta t(F_2 - fu^n) \tag{C.4}$$

2. Substituting (C.3)–(C.4) into (C.1)–(C.2) one has

$$u^{n+1} - f\theta_c\Delta t v^{n+1} = u^* - f\theta_c\Delta t v^n \tag{C.5}$$

$$v^{n+1} + f\theta_c\Delta t u^{n+1} = v^* + f\theta_c\Delta t u^n \tag{C.6}$$

or

$$u^{n+1} = u^* \quad + \quad \frac{f\theta_c\Delta t(\Delta v - f\theta_c\Delta t\Delta u)}{1+(f\theta_c\Delta t)^2} \tag{C.7}$$

$$v^{n+1} = v^* \quad - \quad \frac{f\theta_c\Delta t(\Delta u + f\theta_c\Delta t\Delta v)}{1+(f\theta_c\Delta t)^2} \tag{C.8}$$

where

$$\Delta u = u^* - u^n\,, \quad \Delta v = v^* - v^n \tag{C.9}$$

The following remarks apply

- The forcing terms F_i on the right hand side of (C.1)–(C.2) contain both explicit and implicit terms. A simplification has been made since the latter one are taken at level * and not at the new time t^{n+1}.
- In the 3-D case the new time level t^{n+1} is replaced by the predictor step t^p.
- A four-point spatial interpolation is needed for the evaluation of the currents in the Coriolis terms. Since the open boundary conditions are applied only after solving the momentum equations at all interior nodes, open boundary values are excluded from the interpolation procedure.